1. Consider a simple harmonic oscillator where friction is present.

   In class, we looked at a specific equation of motion: $\ddot{x}(t) + \dot{x}(t) + x(t) = 0$. We showed that it was *under*-damped.

   (a) Determine whether reducing drag by half (i.e. $\ddot{x}(t) + \frac{1}{2}\dot{x}(t) + x(t) = 0$) results in a change in the nature of the solution: is it still under-damped?

   (b) Plot the solution, $x(t)$, given the initial condition $x_0 = 0$, $v_0 = 0$.

   (c) Plot the solution, $x(t)$, given the initial condition $x_0 = 1$, $v_0 = 0$.

   (d) Plot the solution, $x(t)$, given the initial condition $x_0 = 0$, $v_0 = 1$.

   (e) Determine whether doubling the drag coefficient (i.e. $\ddot{x}(t)+2\dot{x}(t)+x(t) = 0$) results in a change in the nature of the solution: is it still under-damped? How do you know?

   (f) Plot the solution, $x(t)$, given the initial condition $x_0 = 0$, $v_0 = 0$.

   (g) Plot the solution, $x(t)$, given the initial condition $x_0 = 1$, $v_0 = 0$.

   (h) Plot the solution, $x(t)$, given the initial condition $x_0 = 0$, $v_0 = 1$.

   (i) Determine whether tripling the drag coefficient (i.e. $\ddot{x}(t) + 3\dot{x}(t) + x(t) = 0$) results in a change in the nature of the solution: is it under-damped? critically-damped? or overdamped?

   (j) Plot the solution, $x(t)$, given the initial condition $x_0 = 0$, $v_0 = 0$.

   (k) Plot the solution, $x(t)$, given the initial condition $x_0 = 1$, $v_0 = 0$.

   (l) Plot the solution, $x(t)$, given the initial condition $x_0 = 0$, $v_0 = 1$.

2. When we initially constructed an equation of motion, we had an equation that looked like $m\ddot{x}(t) = -\alpha\dot{x}(t) + -kx(t)$, where $\alpha > 0$ and $k > 0$. Suppose we had made a typo, and wrote $m\ddot{x}(t) = \alpha\dot{x}(t) + kx(t)$. Provide a rough sketch of what the phase-space diagram would look like. Why is this unphysical?

3. Consider function $x(t) = Ae^{-\beta t}cos(\omega_1 t - \delta)$. As you know, this is a solution for the underdamped case. For the specific case of A=1 cm, $\omega_0 = 1$ rad / s, $\beta = 0.2$ s^{-1}, $m = 1$ kg, and $\delta = \pi/2$ rad, plot the phase diagram for $\dot{x}(t)$ and $x(t)$

4. By this stage, you have shown on more than one occasion that when a force-field is conservative, energy is conserved. You also know that when friction is present, it is *not* conserved. For a simple harmonic oscillator where friction is present (i.e. $\alpha \neq 0$), derive and expression for the total energy as a function of time. Show that in the limit of $t \to \infty$, $E_{\text{total}} = 0$. Does this violate any Laws of Physics? Why not?

5. What is your experience in computer programming? What languages do you know / are comfortable with?