In this assignment, you will continue with the skills you developed in Assignment 8. We will now treat oscillatory systems.

1. Consider a particle of mass m which is connected to a wall by a spring (with spring constant $k = 2.0$ N/m). Assume it has some initial position x_0 and initial velocity, v_0.
   (a) Write down the equation of motion for this system.
   (b) Solve the equation of motion *analytically* (i.e. as we have done before).
   (c) On simulator.science.uoit.ca, make a directory called assignment9
   (d) As with the previous assignment, if we iterate time in small chunks (Δt), it is possible to obtain a numerical solution for $x(t)$ by assuming acceleration is constant over this small interval. Given a value for $v(t)$, we can estimate $v(t + \Delta t) = v(t) + a(t)\Delta t$. Use this approach to numerically solve the problem for a mass = 3.0 kg with x_0=2.0 m (away from the equilibrium position), with an initial velocity $v_0 = 0.0$ m/s. Determine a suitable value for Δt by looking at how well the total energy is conserved in your system. Leave a copy of your source code and executable (with python they are the same file) in your assignment directory. Call it q2.
   (e) Compare your result from (d) with the analytic solution obtained in (b): i.e. produce a plot of $x(t)$ using gnuplot for both.
   (f) Try $v_0 =$ 30.0 m/s. Produce a plot, and again compare your numerical result with the curve produced from the analytic solution.
   (g) Modify your code so that it *calculates* the frequency of oscillation, ω, directly from your numerical result. Compare this value with what you expect for a SHO.

2. We will now include linear (i.e. with respect to velocity) air friction. Copy your source code file to a new file called q3.
   (a) Write down the equation of motion in the presence of air friction with a linear dependance on the velocity.
   (b) Solve the equation of motion assuming an initial position x_0 and initial velocity v_0.
   (c) Modify q3 to include air friction.
   (d) Produce a plot of $x(t)$ assuming $x_0 = 10.0$ m, $v_0 = 30.0$ m/s, and $\alpha = 10.0$ for both the analytic and numerical solution. Does this plot make sense? What do you observe?
   (e) Reduce the drag coefficient by a factor of 100 ($\alpha = 0.1$). Rerun your simulation. Compare this result to the friction free case (produce a plot). Do you get the same answer? Does your result make sense?
   (f) Using what you know about damping from class, determine the values of α corresponding to a system which is underdamped, critically damped, and overdamped. Plot each of these three cases for both the analytical and numerical solution.
   (g) Strictly speaking, it is impossible to define a unique frequency for these damped systems since the system is not periodic with respect to time. We can still try and

quantify the oscillatory nature by considering the time between successive maxima in your curve (for the underdamped case). Modify your code so it estimates a frequency based on this approach.

(h) Generate a p, x phase plot for the underdamped case.

3. We will now consider a position dependent force. Recall the potential from Assignment 5: $\frac{x^4}{4} + \frac{-10x^3}{3} + \frac{31x^2}{2} - 30x$.

   (a) Copy your source code file to a new file called q4.

   (b) Previously, we only solved the EOM for this system when oscillations were small in amplitude and centred around minima. Given your new numerical abilities, we can now treat this system directly. Modify your file q4 so that it can compute the position dependent force generated from the potential above.

   (c) Consider a particle placed at an initial position $\mathrm{x}_0 = 5.9$m. Compute $x(t)$. Explain the behaviour you observe in the context of the shape of the potential.

   (d) Try now $\mathrm{x}_0 = 5.5$m. Again, explain your result.

   (e) Compute the phase-space diagram for part (c) and (d). How are they similar, how are they different?

   (f) Consider a particle placed in the bottom of the lower well. If it has sufficient initial kinetic energy, it can overcome the middle barrier. Can you find an initial kinetic energy which is large enough to overcome this barrier, but small enough that the particle remains in the upper well?

   (g) Modify q4 to include air friction which depends on the *square* of the velocity (assume $\alpha' = 0.1$.). Repeat part (e) and (f)

4. Finally, we will consider the break-down of the small amplitude approximation for a potential you might encounter in Nature. Consider the potential $4\epsilon[(\frac{\sigma}{x})^{12} - (\frac{\sigma}{x})^6]$ where $\epsilon = 10$ and $\sigma = 5$. Known as the Lennard-Jones potential, it is a decent model for a molecular bond. Calculate the oscillation frequency as a function of amplitude for this potential. To visualize the potential (always a good idea), try an x-range of 5 to 10. You may keep the mass at $m = 3.0$.